

Can a resonance theory be a renormalizable theory?

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Abstract. In this talk we make an exhaustive analysis of the possible chiral invariant operators that may describe the resonance decay $S \rightarrow \pi\pi$. These provide at the same time the only available chiral invariant structures for the loop ultraviolet divergences in this amplitude. Independently of the order in perturbation theory, we find just one single-trace term (four if multi-trace operators are allowed), whose renormalization renders the matrix element finite.

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INTRODUCTION

Although there is no argument that allows to affirm that the whole hadronic action is renormalizable, it is possible to prove this for some sectors of the theory [1]¹. The motivation of the present work can be found in the analysis by Rosell *et al.* where the resonance chiral theory one-loop generating functional was calculated [2]. Only chiral Goldstones, scalars and pseudo-scalar resonances were considered in the lagrangian of their approach. All the one-loop ultraviolet (UV) divergences of the theory were computed, finding the corresponding chiral operators required to fulfill the renormalization. However, some new operators that could have been *a priori* expected were not necessary to render the functional finite. In particular, a later work [3] found that, after imposing a vanishing behaviour at high-energies, there were no new UV divergent structures in the one-loop SS-PP correlator². All one needed to make the amplitude finite was a renormalization of the couplings already in the original lagrangian. A similar result was found in a dispersive analysis of two-point Green-functions [4].

These results have provided some clues that may help to understand the way how phenomenological lagrangians must be constructed. Thanks to meson field redefinitions in the generating functional $W[J]$ it is possible to greatly simplify the structure of the hadronic action, with the simplifications occurring at the level of the lagrangian, not of particular amplitudes [2, 5, 6]. Once the operators are removed from the action they are no longer relevant for either on-shell, off-shell, tree-level or loop amplitudes. This is particularly relevant when the calculation is taken to the loop level [2, 6]–[10].

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² Defined as $\Pi(q^2)_{SS-PP} = i \int d^4x e^{iqx} \langle 0 | T \{ J(x)^\dagger J(0) - J_5(x)^\dagger J_5(0) \} | 0 \rangle$, with $J = \bar{q}q'$, $J_5 = i\bar{q}\gamma_5 q'$.

These techniques are applied to the analysis of the scalar meson decay into two Goldstones, $S \rightarrow \pi\pi$, which is found to be described at tree-level by a finite basis of chiral invariant operators. Several important conclusions are extracted, as the fully model-independent description of the $S\pi\pi$ -vertex and the existence of a finite number of local chiral-invariant structures for the UV loop divergences for this amplitude. The chiral limit is assumed all along the work.

A CHIRAL THEORY FOR RESONANCES

Building blocks of the hadronic action

We denote as resonance chiral theory (R χ T) to the most general chiral invariant theory including the Goldstones from the spontaneous chiral symmetry breaking and the mesonic resonances. The recovery of chiral perturbation theory (χ PT) [11]–[13] at low energies requires the R χ T lagrangian to be invariant under chiral transformations. This reduces the number of structures that we can build; in general, just putting meson fields together (e.g. $i\lambda \langle A_{\mu\nu}[V^{\mu\nu}, \pi] \rangle$) does not produce chiral invariant terms (even if they are invariant under $SU(3)_V$) as they may lack of the minimal derivative structure required in the Goldstone interaction.

The building blocks of the theory are the covariant tensors made out of the Goldstone fields and $q\bar{q}$ resonance multiplets (V, A, S, \dots). The Goldstones enter through a non-linear realization that transforms like $(\xi_L(\pi), \xi_R(\pi)) \xrightarrow{g} (g_L \xi_L(\pi) h(g, \pi)^\dagger, g_R \xi_R(\pi) h(g, \pi)^\dagger)$. We choose the canonical coset representatives $\xi_R(\pi) = \xi_L^\dagger(\pi) \equiv u(\pi)$, with the exponential realization $u = \exp\{i\pi/\sqrt{2}F\}$ [13]. Combined together with the external auxiliary fields $J = s, p, \ell^\mu, r^\mu$ it is possible to define the basic tensors

$$\begin{aligned} u_\mu &= i\{u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - i\ell_\mu)u^\dagger\}, \\ \chi_\pm &= u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \\ f_\pm^{\mu\nu} &= u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u, \end{aligned} \tag{1}$$

which transform covariantly in the form

$$X \xrightarrow{g} h X h^\dagger. \tag{2}$$

The field $\chi = 2B_0(s + ip)$ contains the scalar and pseudo-scalar external sources, s and p respectively. The $F_{R,L}^{\mu\nu}$ are the strength-field tensors of the r^μ and ℓ^μ sources [12, 13].

The other ingredients of the theory are the $\bar{q}q$ resonances, which transform linearly as $U(3)_V$ multiplets under the vector subgroup. The variation under a general element of the chiral group is defined by $R \xrightarrow{g} h R h^\dagger$, similar to that in Eq. (2) [14].

In addition to the covariant tensors $X = u^\mu, \chi_\pm, f_\pm^{\mu\nu}, R$, one can construct terms of the form $\nabla^\alpha \dots \nabla^\mu X$, with as many covariant derivatives as desired and also transforming covariantly like in Eq. (2). The covariant derivative is given by [13, 14]

$$\nabla_\mu X = \partial_\mu X + [\Gamma_\mu, X], \tag{3}$$

with the chiral connection $\Gamma_\mu = \frac{1}{2}\{u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - i\ell_\mu)u^\dagger\}$. The commutation $[\nabla_\mu, \nabla_\nu]X = [\Gamma_{\mu\nu}, X]$ is provided by the tensor $\Gamma_{\mu\nu} = \frac{1}{4}[u_\mu, u_\nu] - \frac{i}{2}f_{+\mu\nu}$.

Putting these elements together and taking flavour traces one gets the different chiral-invariant operators for the $R\chi T$ lagrangian [14], e.g, $\langle \nabla_\alpha X_1 \nabla^\alpha \nabla_\mu X_2 \dots \rangle$, $\langle X_1 \rangle \cdot \langle \nabla_\mu X_2 \dots \rangle \dots$. However, symmetry does not impose any constraint on the number of derivatives or resonance fields, it only determines the way how the hadronic fields must be combined [14]–[16]. $\langle \dots \rangle$ is short for trace in the flavour space.

Challenges in the construction of hadronic lagrangians

At the moment of writing down a hadronic description of QCD there is a set of important issues that must be addressed. First, one needs a formal perturbation theory on some parameter that suppresses hadron loops and makes lowest order contributions dominant. The $1/N_C$ expansion based on 't Hooft's large number of colours limit [17] seems to be the most suitable one for a theory with resonances [6], being each meson loop suppressed by a power of $1/N_C$ [18].

The validity of the $1/N_C$ expansion for any energy allows to connect the resonance theory with the high-energy description of QCD, provided by perturbative QCD and the operator product expansion (OPE) [19]. The short-distance matching has produced very successful determinations both at leading order (LO) [20] and at next-to-leading order in $1/N_C$ [6, 8], although the uncertainties in the matching procedure are not yet fully understood [21]. In any case, no high-energy analysis is considered in the present work [1], though it may result fruitful in future studies.

In this talk we rather focus on the implementation of chiral symmetry on the hadronic action. This allows the construction of an infinite number of invariant operators, what may look discouraging. However, one must keep in mind the real goal:

- **The action may contain an infinite number of operators.** This is something already familiar to us through χPT where even at LO we have an infinite number of (related) terms due to the non-linear Goldstone realization.
- **Nonetheless, the crucial point to make the theory predictive is that for a given amplitude at a given order in the established perturbative expansion, only a finite number of operators is required.** This is what happens in the previous example of χPT . Even if there is an infinity of terms, only two operators are required to describe, for instance, the $\pi\pi$ scattering at LO in the chiral expansion.

SIMPLIFYING THE $R\chi T$ LAGRANGIAN

Intuitive picture, formal procedure

Some operators of the $R\chi T$ lagrangian, allowed by the symmetry, are actually redundant and do not carry any extra physical information. An intuitive way to understand this relies on the picture in Fig. (1). The contribution from some operators may look like a non-local meson exchange but, nevertheless, they always appear through local structures. Due to the form of the vertex (for instance, $\lambda \langle \dots (\nabla^2 + M_\pi^2) S \rangle$), the intermediate

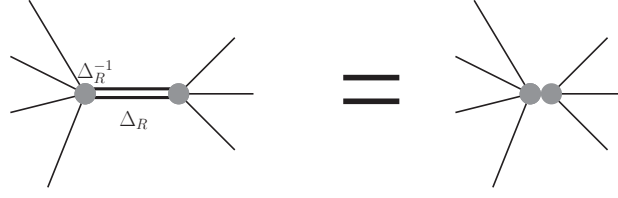


FIGURE 1. Local structure in non-local meson exchanges.

propagator is canceled out any time this vertex enters into play, resembling a local operator contribution. Since $R\chi T$ includes all the operators compatible with the symmetry, the structure on the right-hand side of Fig. (1) is already contained in the lagrangian.

The formal procedure to remove these redundant terms relies on the freedom to perform meson field redefinitions in the generating functional $W[J]$ [5]. This transforms some operators into others and, if it is conveniently tuned, it is possible to fully remove the undesired terms.

Goldstone and scalar resonance transformations

The starting point is the $R\chi T$ lagrangian, with the completely general structure [1]

$$\mathcal{L} = \frac{F_0^2}{4} \langle u_\mu u^\mu \rangle + \langle A_S \nabla^\mu u_\mu \rangle + \langle B_S \rangle - \frac{1}{2} \langle S(\nabla^2 + M_S^2)S \rangle + \Delta\mathcal{L}, \quad (4)$$

where $\Delta\mathcal{L}$ is not relevant in the present study and we just provide its general structure:

$$\Delta\mathcal{L} = \mathcal{O}(S^2 u^\alpha u^\beta) + \mathcal{O}(S^3) + \mathcal{O}(R') + \mathcal{O}(J) + \mathcal{O}(u^\alpha u^\beta u^\mu u^\nu), \quad (5)$$

with the term $\mathcal{O}(R')$ containing at least one resonance $R' \neq S$. $\langle A_S \nabla^\mu u_\mu \rangle$ and $\langle B_S \rangle$ account for all the operators made out of just one S -meson field and two tensors u^α , but allowing any number of covariant derivatives: $A_S \nabla^\mu u_\mu$, $B_S \sim S \nabla \dots u^\alpha \nabla \dots u^\beta$.

We will perform first a Goldstone field redefinition that induces a shift in u^μ of the form $u_\mu \longrightarrow u_\mu + \frac{2}{F_0^2} \nabla_\mu A_S + \mathcal{O}(A_S^2)$ [1]. The required Goldstone transformation is not unique, being one of the simplest $\xi_R \rightarrow \xi_R \exp\{-iA_S/F_0^2\}$, $\xi_L \rightarrow \xi_L \exp\{iA_S/F_0^2\}$. This produces a lagrangian with exactly the same structure as in Eqs. (4)–(5) except for the term $\langle A_S \nabla^\mu u_\mu \rangle$, which is completely removed.

The second step relies on the scalar resonance field transformation. The remnant term $\langle B_S \rangle$ is decomposed in the form,

$$\langle B_S \rangle = \langle \zeta (\nabla^2 + M_S^2) S \rangle + \langle \eta S \rangle, \quad (6)$$

where ζ and η are local chiral tensors containing just Goldstones. At this point it is easy to realise that the change $S \rightarrow S + \zeta$ removes the first term on the right-hand side of Eq. (6), leaving the finally simplified lagrangian,

$$\mathcal{L} = \frac{F_0^2}{4} \langle u^\mu u_\mu \rangle + \langle \eta S \rangle - \frac{1}{2} \langle S(\nabla^2 + M_S^2)S \rangle + \Delta\mathcal{L}, \quad (7)$$

where all the operator that could be written like $\langle A_S \nabla^\mu u_\mu \rangle$ or $\langle \zeta (\nabla^2 + M_S^2) S \rangle$ have been removed.

$S \rightarrow \pi\pi$ decay amplitude

In the construction of the most general form for chiral invariant operators contributing to $S \rightarrow \pi\pi$ we have to take into account that, in the chiral limit, we cannot include the tensors $\chi_\pm, f_\pm^{\mu\nu}$ since they are proportional to external sources. We must include one S field and exactly two tensors u^α . Otherwise, the operator does not preserve parity or it produces more than two Goldstones in the final state. No *a priori* restriction can be made on the number of covariant derivatives. In the chiral limit, this gives the general form

$$\mathcal{L}_{S \rightarrow \pi\pi} = \lambda \langle S \{ \nabla^{\mu_1} \dots \nabla^{\mu_m} u^\rho, \nabla^{\nu_1} \dots \nabla^{\nu_n} u^\sigma \} \rangle \times t_{\mu_1, \dots, \mu_m, \rho, \nu_1, \dots, \nu_n, \sigma}, \quad (8)$$

where the Lorentz tensor $t_{\mu_1, \dots, \mu_m, \rho, \nu_1, \dots, \nu_n, \sigma}$ handles all the possible contractions of the indices. The anticommutator $\{ \dots, \dots \}$ ensures that the operator is invariant under charge and hermitian conjugations [13].

The simplest operator of this kind is the familiar term,

$$\mathcal{L}_{S \rightarrow \pi\pi} = \lambda \langle S \{ u^\mu, u_\mu \} \rangle = 2\lambda \langle S u^\mu u_\mu \rangle, \quad (9)$$

which is just the $c_d \langle S u^\mu u_\mu \rangle$ operator in Ref. [14].

For a higher number of derivatives, one has different possible contractions of the Lorentz indices. The detailed analysis of the different cases is done in Ref. [1], where it is concluded that these operators either show the structure $\langle A_S \nabla^\mu u_\mu \rangle$, or the form $\langle \zeta (\nabla^2 + M_S^2) S \rangle$, or they are equivalent to an operator with two derivatives less. The first two correspond to operators that can be fully removed through field redefinitions and the third one allows to iteratively simplify the operator and to reduce it into the c_d term in Eq. (9).

If multitrace operators -subleading in $1/N_C$ - are allowed then there are another three independent operators: $\lambda_a \langle S \rangle \langle u_\mu u^\mu \rangle$, $\lambda_b \langle S u_\mu \rangle \langle u^\mu \rangle$ and $\lambda_c \langle S \rangle \langle u_\mu \rangle \langle u^\mu \rangle$, exhausting the list of chiral invariant operators contributing to the decay $S \rightarrow \pi\pi$.

FINITE BASIS AND RENORMALIZABILITY

This provides a clear example of the possibility of constructing a fully model independent lagrangian for the description of hadronic processes. As it has been noted, the action may contain an infinite number of operators but the $S \rightarrow \pi\pi$ amplitude is given at large- N_C by just the c_d term.

What implications does this have on the renormalizability of the amplitude? The only available local chiral invariant structures for the UV divergences appearing in the $S \rightarrow \pi\pi$ decay at the loop level are these four operators $O_{c_d}, O_{\lambda_a}, O_{\lambda_b}, O_{\lambda_c}$. Therefore, the renormalization of the four couplings $c_d, \lambda_a, \lambda_b, \lambda_c$ renders the amplitude finite at any order in perturbation theory.

Preliminary studies have found similar simplifications in a wider set of amplitudes, which are also described at tree-level by a finite number of independent operators [22]. We plan to extend the analysis to other S -meson processes and amplitudes with other resonances. It can be also applied to the heavy quark meson sector and to the study of Green-functions.

The possibility of this to be a general feature of the lagrangian is more difficult to defend although it would lead to very deep implications. If an amplitude \mathcal{M} at any order in perturbation theory is given at tree-level by a fixed and finite number N of chiral invariant operators ($S[\phi] = \dots + \sum_{k=1}^N \int dx^D c_k O_k[\phi, J]$) then the local UV divergences can only show this structure and the generating functional has then the form

$$W[J] = \dots + \sum_{k=1}^N \int dx^D c_k O_k[\phi^{cl}, J] + \sum_{k=1}^N \int dx^D \lambda_\infty \Gamma_k O_k[\phi^{cl}, J] , \quad (10)$$

with λ_∞ containing the UV divergence. Thus, the renormalization of the couplings c_k would render the amplitude finite at any order in perturbation theory.

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